Belief Logic

u:A	=	You believe that A.
	=	You accept A.
$\underline{u}$:A	=	Believe that A.
	=	Accept A.

1. The result of writing a small letter and then ":" and then a wff is a *descriptive* wff.
2. The result of writing an underlined small letter and then ":" and then a wff is an *imperative* wff.

You believe that A is true	=	u:A
You don't believe that A is true	=	~u:A
You believe that A is false	=	u:~A
You don't believe A and you don't believe not-A	=	(~u:A • ~u:~A)
You believe that you ought to do A	=	u:OA<u>u</u>
Everyone believes that they ought to do A	=	(x)x:OA<u>x</u>
You believe that if A then not-B	=	u:(A ⊃ ~B)
If you believe A, then you don't believe B	=	(u:A ⊃ ~u:B)

Believe that A is true	=	$\underline{u}$:A
Don’t believe that A is true	=	~$\underline{u}$:A
Believe that A is false	=	$\underline{u}$:~A
Don’t believe A and don’t believe not-A	=	(~$\underline{u}$:A • ~$\underline{u}$:~A)
Believe that you ought to do A	=	$\underline{u}$:OA$\underline{u}$
Let everyone believe that they ought to do A	=	(x)$\underline{x}$:OA$\underline{x}$
If you in fact believe A, then don’t believe B	=	(u:A ⊃ ~$\underline{u}$:B)
Don’t combine believing A with believing B	=	~($\underline{u}$:A • $\underline{u}$:B)

Three Approaches to Belief Logic

1. Belief logic studies what belief formulas validly follow from what other belief formulas.
2. Belief logic studies how people would believe if they were *completely consistent believers*.
3. Belief logic generates *consistency imperatives*, like:
   - "Don't combine believing A with believing not-A"
     "$\sim(\underline{u}:A \cdot \underline{u}:\sim A)$"
   - "Don't combine believing A-and-B with not believing A"
     "$\sim(\underline{u}:(A \cdot B) \cdot \sim\underline{u}: A)$"

Belief worlds are represented by strings of one or more instances of a small letter – like "u," "uu," "uuu," and so on.

A *belief policy* is a set of imperatives about what you are or are not to believe, e.g., u̲:P, ~u̲:W, ~u̲:~W ("Believe that Michigan will play; be neutral about whether Michigan will win").

Belief logic forbids belief policies that tell you to believe inconsistently (where the set S of things you're told to believe is inconsistent or else S logically entails something that you're told not to believe).

Rule B+ lets you put what you're told to believe into *all* belief worlds. Rule B- lets you put the denial of something you're told not to believe into *some* belief world (do B- first). A belief policy is forbidden if its denial leads to a belief world being inconsistent.

“Don’t combine believing A with believing not-A.”

[$\therefore \sim(\underline{u}{:}A \cdot \underline{u}{:}\sim A)$ **Valid**

* 1 ┌ asm: $(\underline{u}{:}A \cdot \underline{u}{:}\sim A)$
2 │ $\therefore \underline{u}{:}A$ {from 1}
3 │ $\therefore \underline{u}{:}\sim A$ {from 1}
4 │ $u \therefore A$ {from 2} ⇦
5 └ $u \therefore \sim A$ {from 3} ⇦
6 $\therefore \sim(\underline{u}{:}A \cdot \underline{u}{:}\sim A)$ {from 1; 4 contradicts 5}

Apply B- before B+

B- If you’re told to *refrain* from believing A, then put not-A in a *new* belief world of yours.

B+ If you’re told to believe A, then put A in all of your belief worlds.

“Don’t combine believing A-and-B with not believing A.”

[∴ ~($\underline{u}$:(A • B) • ~$\underline{u}$:A) **Valid**

* 1 ┌ asm: ($\underline{u}$:(A • B) • ~$\underline{u}$:A)

2 │ ∴ $\underline{u}$:(A • B) {from 1}

* 3 │ ∴ ~$\underline{u}$:A {from 1}

4 │ u ∴ ~A {from 3} ⇦

5 │ u ∴ (A • B) {from 2}

6 └ u ∴ A {from 5} ⇦

7 ∴ ~($\underline{u}$:(A • B) • ~$\underline{u}$:A) {from 1; 4 contradicts 6}

Apply B- before B+

B- If you’re told to *refrain* from believing A, then put not-A in a *new* belief world of yours.

B+ If you’re told to believe A, then put A in all of your belief worlds.

Belief Inference Rules

B- $\sim\underline{u}:A \rightarrow u \therefore \sim A$, use a *new* string of u's

* First drop negative imperative belief operators; use a new belief world each time.

B+ $\underline{u}:A \rightarrow u \therefore A$, use any string of u's

Then drop positive imperative belief operators; use old belief worlds if you have them (otherwise use a new world "u").

1 ☐(A ⊃ B) Valid

[∴ ~($\underline{u}$:A • ~$\underline{u}$:B)

* 2 ┌ asm: ($\underline{u}$:A • ~$\underline{u}$:B)

3 │ ∴ $\underline{u}$:A {from 2}

* 4 │ ∴ ~$\underline{u}$:B {from 2}

5 │ u ∴ ~B {from 4} ← using B-

6 │ u ∴ A {from 3} ← using B+

* 7 │ u ∴ (A ⊃ B) {from 1}

8 └ u ∴ B {from 6 and 7}

9 ∴ ~($\underline{u}$:A • ~$\underline{u}$:B) {from 2; 5 contradicts 8}

1. Reverse squiggles (quantificational/modal/deontic).
2. Drop weak operators, using new things: ~$\underline{u}$: R (∃x) ◇
3. Lastly, drop strong operators, using old things (if you have them): $\underline{u}$: O (x) ☐

	$u{:}A$	=	You accept (endorse, assent to, say in your heart) “A is true.”
		=	You believe that A.
	$u{:}\underline{A}$	=	You accept (endorse, assent to, say in your heart) “Let act A be done.”
		=	You will that act A be done.
If A is present:	$u{:}A\underline{u}$	=	You accept the imperative for you to do A now.
		=	You act (in order) to do A.
If A is future:	$u{:}A\underline{u}$	=	You accept the imperative for you to do A in the future.
		=	You’re resolved to do A.
If $u \neq x$:	$u{:}A\underline{x}$	=	You accept the imperative for X to do A.
		=	You desire (or want) that X do A.

$u{:}A\underline{u}$	=	You act (in order) to do A.
	=	You say in your heart, “Do A” (addressed to yourself).

Au	=	You do A.

$u{:}(\exists x)(Kx \cdot R\underline{x})$
= You desire that some who kill *repent*.
= You say in your heart “Would that some who kill *repent*.”

$u{:}(\exists x)(K\underline{x} \cdot Rx)$
= You desire that some *kill* who repent.
= You say in your heart “Would that some *kill* who repent.”

$u{:}(\exists x)(K\underline{x} \cdot R\underline{x})$
= You desire that some both *kill* and *repent*.
= You say in your heart “Would that some *kill* and *repent*.”

	<u>u</u>:<u>A</u>	=	Accept (endorse, assent to, say in your heart) "Let act A be done."
		=	Will that act A be done.
If A is present:	<u>u</u>:A<u>u</u>	=	Accept the imperative for you to do A now.
		=	Act (in order) to do A.
If A is future:	<u>u</u>:A<u>u</u>	=	Accept the imperative for you to do A in the future.
		=	Be resolved to do A.
If $u \neq x$:	<u>u</u>:A<u>x</u>	=	Accept the imperative for X to do A.
		=	Desire (or want) that X do A.

Use underlining *before* “:” to *tell* someone what to believe or will.

Use underlining *after* “:” if the sentence is about *willing*.

Indicatives			*Imperatives*		
u:A	=	You believe A.	$\underline{u}$:A	=	Believe A.
u:$\underline{A}$	=	You will A.	$\underline{u}$:$\underline{A}$	=	Will A.

Don’t combine *believing* that it’s wrong for you to do A with *acting* to do A.

$[\therefore \sim(\underline{u}{:}O{\sim}A\underline{u} \cdot \underline{u}{:}A\underline{u})$ Valid

* 1 ┌ asm: $(\underline{u}{:}O{\sim}A\underline{u} \cdot \underline{u}{:}A\underline{u})$

2 │ $\therefore \underline{u}{:}O{\sim}A\underline{u}$ {from 1}

3 │ $\therefore \underline{u}{:}A\underline{u}$ {from 1}

4 │ $u \therefore O{\sim}A\underline{u}$ {from 2}

5 │ $u \therefore A\underline{u}$ {from 3}

6 └ $u \therefore {\sim}A\underline{u}$ {from 4}

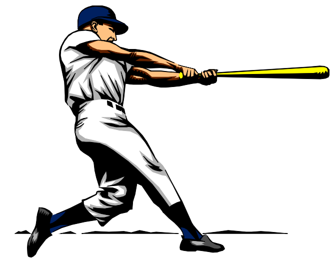
7 $\therefore \sim(\underline{u}{:}O{\sim}A\underline{u} \cdot \underline{u}{:}A\underline{u})$ {from 1; 5 contradicts 6}

	=	A is evident to you.
O<u>u</u>:A	=	It's obligatory (rationally required) that you believe A.
	=	Insofar as intellectual considerations are concerned (including your experiences), you ought to believe A.

	=	A is reasonable for you to believe.
R<u>u</u>:A	=	It's all right (rationally permissible) that you believe A.
	=	Insofar as intellectual considerations are concerned (including your experiences), it would be all right for you to believe A.

It would be unreasonable for you to believe A	=	~Ru:A
It's obligatory that you not believe A	=	O~u:A
It would be reasonable for you to take no position on A	=	R(~u:A • ~u:~A)
It's evident to you that if A then B	=	Ou:(A ⊃ B)
If it's evident to you that A, then it's evident to you that B	=	(Ou:A ⊃ Ou:B)
You ought not to combine believing A with believing not-A	=	O~(u:A • u:~A)

knowledge	=	*evident true belief* [roughly]
You know that A	=	A is evident to you, A is true, & you believe A.
uKA	=	(Ou:A • (A • u:A))



Hub	=	You hit the ball.
Hu̲b	=	Hit the ball.
OHu̲b	=	You ought to hit the ball.
RHu̲b	=	It’s all right for you to hit the ball.
u:Hub	=	You believe that you’ll hit the ball.
u̲:Hub	=	Believe that you’ll hit the ball.
u:Hu̲b	=	You act (with the intention) to hit the ball.
u̲:Hu̲b	=	Act (with the intention) to hit the ball.
Ou̲:Hub	=	You ought to believe (insofar as your evidence goes) that you’ll hit the ball = It’s evident to you that you’ll hit the ball.
Ru̲:Hub	=	It’s all right (reasonable) for you to believe that you’ll hit the ball (insofar as your evidence goes).

1 O$\underline{u}$:G — Valid

[∴ ~R$\underline{u}$:~G

* 2 ┌ asm: R$\underline{u}$:~G

3 │ D ∴ $\underline{u}$:~G {from 2}

4 │ D ∴ $\underline{u}$:G {from 1}

5 │ Du ∴ ~G {from 3} ← using B+

6 └ Du ∴ G {from 4} ← using B+

7 ∴ ~R$\underline{u}$:~G {from 2; 5 contradicts 6}

Theism is evident for you.

∴ Atheism is unreasonable for you.

1. Reverse squiggles (quantificational/modal/deontic).
2. Drop weak operators, using new things: ~$\underline{u}$: R (∃x) ◇
3. Lastly, drop strong operators, using old things (if you have them): $\underline{u}$: O (x) □

		[∴ O~($\underline{u}$:O~A$\underline{u}$ • $\underline{u}$:A$\underline{u}$)	Valid
*	1	asm: ~O~($\underline{u}$:O~A$\underline{u}$ • $\underline{u}$:A$\underline{u}$)	
*	2	∴ R($\underline{u}$:O~A$\underline{u}$ • $\underline{u}$:A$\underline{u}$)	{from 1}
*	3	D ∴ ($\underline{u}$:O~A$\underline{u}$ • $\underline{u}$:A$\underline{u}$)	{from 2}
	4	D ∴ $\underline{u}$:O~A$\underline{u}$	{from 3}
	5	D ∴ $\underline{u}$:A$\underline{u}$	{from 3}
	6	Du ∴ O~A$\underline{u}$	{from 4} ← using B+
	7	Du ∴ A$\underline{u}$	{from 5} ← using B+
	8	Du ∴ ~A$\underline{u}$	{from 6}
	9	∴ O~($\underline{u}$:O~A$\underline{u}$ • $\underline{u}$:A$\underline{u}$)	{from 1; 7 contra 8}

You ought not to combine *believing* that it's wrong for you to do A with *acting* to do A.

1. Reverse squiggles (quantificational/modal/deontic).
2. Drop weak operators, using new things: ~$\underline{u}$: R (∃x) ◇
3. Lastly, drop strong operators, using old things (if you have them): $\underline{u}$: O (x) ☐

Our belief logic is oversimplified in three ways. A more sophisticated belief logic would:

- add qualifications to the implicit “One ought to be consistent” axiom and the derived consistency norms,
- perhaps qualify the conjunctivity principle (because of the lottery paradox), and
- add a second deontic operator O* (for what one ought to believe insofar as intellectual considerations go) distinct from O (for what we ought to do all-things-considered).

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