Imperative Logic

Indicative (You're doing A.)
A Au

Imperative (Do A.)
$\underline{A}$ $A\underline{u}$

1. Any underlined capital letter is a wff.
2. The result of writing a capital letter and then one or more small letters, one small letter of which is underlined, is a wff.

Don’t do A = $\sim\underline{A}$
Do A and B = $(\underline{A} \cdot \underline{B})$
Do A or B = $(\underline{A} \vee \underline{B})$
Don’t do either A or B = $\sim(\underline{A} \vee \underline{B})$

Don’t both do A and do B =
Don’t combine doing A with doing B = $\sim(\underline{A} \cdot \underline{B})$

Don’t combine doing A with not doing B =
Don’t do A without doing B = $\sim(\underline{A} \cdot \sim\underline{B})$

You’re doing A and you’re doing B = $(A \cdot B)$
You’re doing A, but do B = $(A \cdot \underline{B})$
Do A and B = $(\underline{A} \cdot \underline{B})$

If you’re doing A, then you’re doing B = $(A \supset B)$
If you (in fact) are doing A, then do B = $(A \supset \underline{B})$
Do A, only if you (in fact) are doing B = $(\underline{A} \supset B)$

If you (in fact) are doing A, then don’t do B = $(A \supset {\sim}\underline{B})$
Don’t combine doing A with doing B = ${\sim}(\underline{A} \cdot \underline{B})$

X, do (or be) A = $A\underline{x}$

X, do A to Y = $A\underline{x}y$

Everyone does A = $(x)Ax$

Let everyone do A = $(x)A\underline{x}$

Let everyone who (in fact) is doing A do B = $(x)(Ax \supset B\underline{x})$

Let someone who (in fact) is doing A do B = $(\exists x)(Ax \cdot B\underline{x})$

Let someone both do A and do B = $(\exists x)(A\underline{x} \cdot B\underline{x})$

Imperative Arguments

	If the cocoa is about to boil, remove it from the heat.	$(B \supset \underline{R})$ Valid
	The cocoa is about to boil.	B
∴	Remove it from the heat.	∴ $\underline{R}$

- Use the same inference rules as before; but treat "A" and "$\underline{A}$" as different wffs.
- An argument is VALID if it is inconsistent to join the premises with the contradictory of the conclusion.
- Alternatively, VALID = if the premises are correct ("1") then so must be the conclusion.

Don't combine accelerating with braking.
You're accelerating.
∴ Don't brake.

$*$ 1 $\sim(\underline{A}^{0} \cdot \underline{B}^{1}) = 1$
2 $A^{1} = 1$
[$\therefore \sim\underline{B}^{1} = 0$
3 asm: $\underline{B}$
4 $\therefore \sim\underline{A}$ {from 1 and 3}

Invalid

$\boxed{A, \sim\underline{A}, \underline{B}}$

On our refutation:

$A = 1$
$\underline{A} = 0$
$\underline{B} = 1$

This is consistent:

You're accelerating.
Don't accelerate.
Instead, brake.

	Don't combine accelerating with braking.	$\sim(\underline{A} \cdot \underline{B})$	Invalid
	You're accelerating.	A	
∴	Don't brake.	∴ $\sim\underline{B}$	

	If you're accelerating, then don't brake.	$(A \supset \sim\underline{B})$	Valid
	You're accelerating.	A	
∴	Don't brake.	∴ $\sim\underline{B}$	

$(A \supset \sim\underline{B})$	=	If you do A, then don't believe that A is wrong.
$(B \supset \sim\underline{A})$	=	If you believe that A is wrong, then don't do A.
$\sim(\underline{B} \cdot \underline{A})$	=	Don't combine believing that A is wrong with doing A.

Deontic Logic

Indicative (You're doing A.)	*Imperative* (Do A.)	*Deontic* (You ought to do A.)
A	$\underline{A}$	$O\underline{A}$
Au	$A\underline{u}$	$OA\underline{u}$

3. The result of writing "O" or "R," and then an imperative wff, is a deontic wff.

$O\underline{A}$	=	It's obligatory that A.
$OA\underline{x}$	=	X ought to do A.
$OA\underline{x}y$	=	X ought to do A to Y.

$R\underline{A}$	=	It's permissible that A.
$RA\underline{x}$	=	It's all right for X to do A.
$RA\underline{x}y$	=	It's all right for X to do A to Y.

Act A is wrong	=	$\sim R\underline{A}$	=	Act A isn't all right.
	=	$O\sim\underline{A}$	=	Act A ought not to be done.

It ought to be that A and B	=	$O(\underline{A} \cdot \underline{B})$
It's all right that A or B	=	$R(\underline{A} \vee \underline{B})$

If you do A, then you ought not to do B	=	$(A \supset O{\sim}\underline{B})$
You ought not to combine doing A with doing B	=	$O{\sim}(\underline{A} \cdot \underline{B})$

It's obligatory that everyone do A	=	$O(x)A\underline{x}$
It isn't obligatory that everyone do A	=	${\sim}O(x)A\underline{x}$
It's obligatory that not everyone do A	=	$O{\sim}(x)A\underline{x}$
It's obligatory that everyone refrain from doing A	=	$O(x){\sim}A\underline{x}$

It's obligatory that someone answer the phone = $O(\exists x)A\underline{x}$

There's someone who has the obligation to answer the phone = $(\exists x)OA\underline{x}$

It's obligatory that some who kill repent = $O(\exists x)(Kx \cdot R\underline{x})$

It's obligatory that some kill who repent = $O(\exists x)(K\underline{x} \cdot Rx)$

It's obligatory that some both kill and repent = $O(\exists x)(K\underline{x} \cdot R\underline{x})$

Deontic Proofs

- A *world prefix* is a string of zero or more instances of "W" or "D."
- A *possible world* is a consistent and complete set of indicatives and imperatives.
- A *deontic world* is a possible world in which the indicative statements are all true and the imperatives prescribe some jointly permissible combination of actions.
- "O$\underline{A}$" is true if and only if "$\underline{A}$" is in *all* deontic worlds.
- "R$\underline{A}$" is true if and only if "$\underline{A}$" is in *some* deontic worlds.

Suppose that these indicatives are all true:

- I have an 8 am class.
- I ought to get up before 7 am.
- It would be permissible for me to get up at 6:45 am.
- It would be permissible for me to get up at 6:30 am.

Then deontic worlds D and DD might contain these, in addition to the indicatives listed above:

D	Get up before 7 am. Get up at 6:45 am.
DD	Get up before 7 am. Get up at 6:30 am.

Deontic Inference Rules

First reverse squiggles	~O$\underline{A}$ → R~$\underline{A}$ ~R$\underline{A}$ → O~$\underline{A}$	*
and drop R's;	R$\underline{A}$ → D ∴ $\underline{A}$, use a *new* string of D's	*
lastly, drop O's.	O$\underline{A}$ → D ∴ $\underline{A}$, use a blank or any string of D's	Don't star

Indicative transfer

$D \therefore A \quad \rightarrow \quad A$,
the world prefixes of the derived and deriving steps must be identical except that one ends in one or more additional D's

We can transfer indicatives freely between a deontic world and whatever world it depends on.

Kant's Law

$O\underline{A} \quad \rightarrow \quad \Diamond A$

"Ought" implies "can":

"You ought to do A" entails "It's possible for you to do A."

Hare’s Law: An “ought” entails the corresponding imperative.

Kant’s Law: “Ought” implies “can.”

Hume’s Law: You can’t deduce an “ought” from an “is.”

Poincaré’s Law: You can’t deduce an imperative from an “is.”

Valid

*	1	R($\underline{A} \cdot \underline{B}$)	
		[∴ R$\underline{A}$	
*	2	asm: ~R$\underline{A}$	
	3	∴ O~$\underline{A}$ {from 2}	← reverse squiggles
*	4	D ∴ ($\underline{A} \cdot \underline{B}$) {from 1}	← drop "R"
	5	D ∴ $\underline{A}$ {from 4}	
	6	D ∴ $\underline{B}$ {from 4}	
	7	D ∴ ~$\underline{A}$ {from 3}	← drop "O"
	8	∴ R$\underline{A}$ {from 2; 5 contradicts 7}	

1. Reverse squiggles.
2. Drop each initial "R," using a new deontic world each time.
3. Lastly, drop each initial "O" once for each old deontic world. (Never use a new deontic world when you drop "O.")

$[\therefore \Box(O(\underline{A} \cdot \underline{B}) \supset O\underline{A})$ Valid

* 1 asm: $\sim\Box(O(\underline{A} \cdot \underline{B}) \supset O\underline{A})$

* 2 $\therefore \Diamond\sim(O(\underline{A} \cdot \underline{B}) \supset O\underline{A})$ {from 1}

* 3 $W \therefore \sim(O(\underline{A} \cdot \underline{B}) \supset O\underline{A})$ {from 2}

4 $W \therefore O(\underline{A} \cdot \underline{B})$ {from 3}

* 5 $W \therefore \sim O\underline{A}$ {from 3}

* 6 $W \therefore R\sim\underline{A}$ {from 5}

7 $WD \therefore \sim\underline{A}$ {from 6}

8 $WD \therefore (\underline{A} \cdot \underline{B})$ {from 4}

9 $WD \therefore \underline{A}$ {from 8}

10 $\therefore \Box(O(\underline{A} \cdot \underline{B}) \supset O\underline{A})$ {from 1; 7 contradicts 9}