Imperative Logic

Indicative (You're doing A.)
A Au

Imperative (Do A.)
$\underline{A}$ $A\underline{u}$

1. Any underlined capital letter is a wff.
2. The result of writing a capital letter and then one or more small letters, one small letter of which is underlined, is a wff.

Don't do A $= \sim\underline{A}$

Do A and B $= (\underline{A} \cdot \underline{B})$

Do A or B $= (\underline{A} \vee \underline{B})$

Don't do either A or B $= \sim(\underline{A} \vee \underline{B})$

Don't both do A and do B $=$
Don't combine doing A with doing B $=$
$\sim(\underline{A} \cdot \underline{B})$

Don't combine doing A with not doing B $=$
Don't do A without doing B $=$
$\sim(\underline{A} \cdot \sim\underline{B})$

You're doing A and you're doing B = $(A \cdot B)$

You're doing A, but do B = $(A \cdot \underline{B})$

Do A and B = $(\underline{A} \cdot \underline{B})$

If you're doing A, then you're doing B = $(A \supset B)$

If you (in fact) are doing A, then do B = $(A \supset \underline{B})$

Do A, only if you (in fact) are doing B = $(\underline{A} \supset B)$

If you (in fact) are doing A, then don't do B = $(A \supset \sim\underline{B})$

Don't combine doing A with doing B = $\sim(\underline{A} \cdot \underline{B})$

X, do (or be) A = $A\underline{x}$

X, do A to Y = $A\underline{x}y$

Everyone does A = $(x)Ax$

Let everyone do A = $(x)A\underline{x}$

Let everyone who (in fact) is doing A do B = $(x)(Ax \supset B\underline{x})$

Let someone who (in fact) is doing A do B = $(\exists x)(Ax \cdot B\underline{x})$

Let someone both do A and do B = $(\exists x)(A\underline{x} \cdot B\underline{x})$

Imperative Arguments

If the cocoa is about to boil,
remove it from the heat.
The cocoa is about to boil.
∴ Remove it from the heat.

$(B \supset \underline{R})$ Valid
B
$\therefore \underline{R}$

- Use the same inference rules as before; but treat "A" and "$\underline{A}$" as different wffs.
- An argument is VALID if it is inconsistent to join the premises with the contradictory of the conclusion.
- Alternatively, VALID = if the premises are correct ("1") then so must be the conclusion.

Don’t combine accelerating
with braking.
You’re accelerating.
∴ Don’t brake.

*	1	$\sim(\underline{A}^0 \cdot \underline{B}^1) = 1$	
	2	$A^1 = 1$	
		$[\therefore \sim\underline{B}^1 = 0$	
	3	asm: $\underline{B}$	
	4	$\therefore \sim\underline{A}$	{from 1 and 3}

Invalid

$\boxed{A, \sim\underline{A}, \underline{B}}$

On our refutation:

$A = 1$
$\underline{A} = 0$
$\underline{B} = 1$

This is consistent:

You’re accelerating.
Don’t accelerate.
Instead, brake.

	Don’t combine accelerating with braking.	$\sim(\underline{A} \cdot \underline{B})$	Invalid
	You’re accelerating.	A	
∴	Don’t brake.	∴ $\sim\underline{B}$	
	If you’re accelerating, then don’t brake.	$(A \supset \sim\underline{B})$	Valid
	You’re accelerating.	A	
∴	Don’t brake.	∴ $\sim\underline{B}$	

$(A \supset \sim\underline{B})$ = If you do A, then don’t believe that A is wrong.

$(B \supset \sim\underline{A})$ = If you believe that A is wrong, then don’t do A.

$\sim(\underline{B} \cdot \underline{A})$ = Don’t combine believing that A is wrong with doing A.

Deontic Logic

Indicative (You're doing A.)	*Imperative* (Do A.)	*Deontic* (You ought to do A.)
A Au	$\underline{\text{A}}$ A$\underline{\text{u}}$	O$\underline{\text{A}}$ OA$\underline{\text{u}}$

3. The result of writing "O" or "R," and then an imperative wff, is a deontic wff.

$O\underline{A}$	=	It's obligatory that A.
$OA\underline{x}$	=	X ought to do A.
$OA\underline{x}y$	=	X ought to do A to Y.

$R\underline{A}$	=	It's permissible that A.
$RA\underline{x}$	=	It's all right for X to do A.
$RA\underline{x}y$	=	It's all right for X to do A to Y.

Act A is wrong	=	$\sim R\underline{A}$	=	Act A isn't all right.
	=	$O\sim\underline{A}$	=	Act A ought not to be done.

It ought to be that A and B	=	$O(\underline{A} \cdot \underline{B})$
It's all right that A or B	=	$R(\underline{A} \vee \underline{B})$

If you do A, then you ought not to do B	=	$(A \supset O{\sim}\underline{B})$
You ought not to combine doing A with doing B	=	$O{\sim}(\underline{A} \cdot \underline{B})$

It's obligatory that everyone do A	=	$O(x)A\underline{x}$
It isn't obligatory that everyone do A	=	${\sim}O(x)A\underline{x}$
It's obligatory that not everyone do A	=	$O{\sim}(x)A\underline{x}$
It's obligatory that everyone refrain from doing A	=	$O(x){\sim}A\underline{x}$

It's obligatory that someone answer the phone	=	$O(\exists x)A\underline{x}$
There's someone who has the obligation to answer the phone	=	$(\exists x)OA\underline{x}$

It's obligatory that some who kill repent	=	$O(\exists x)(Kx \cdot R\underline{x})$
It's obligatory that some kill who repent	=	$O(\exists x)(K\underline{x} \cdot Rx)$
It's obligatory that some both kill and repent	=	$O(\exists x)(K\underline{x} \cdot R\underline{x})$

Deontic Proofs

- A *world prefix* is a string of zero or more instances of "W" or "D."
- A *possible world* is a consistent and complete set of indicatives and imperatives.
- A *deontic world* is a possible world in which the indicative statements are all true and the imperatives prescribe some jointly permissible combination of actions.
- "O$\underline{\text{A}}$" is true if and only if "$\underline{\text{A}}$" is in *all* deontic worlds.
- "R$\underline{\text{A}}$" is true if and only if "$\underline{\text{A}}$" is in *some* deontic worlds.

Deontic Inference Rules

First reverse squiggles	$\sim O\underline{A} \rightarrow R\sim\underline{A}$ $\sim R\underline{A} \rightarrow O\sim\underline{A}$	*
and drop R's;	$R\underline{A} \rightarrow D \therefore \underline{A}$, use a *new* string of D's	*
lastly, drop O's.	$O\underline{A} \rightarrow D \therefore \underline{A}$, use a blank or any string of D's	Don't star

Indicative transfer	$D \therefore A \quad \rightarrow \quad A$, the world prefixes of the derived and deriving steps must be identical except that one ends in one or more additional D's	We can transfer indicatives freely between a deontic world and whatever world it depends on.
Kant's Law	$O\underline{A} \quad \rightarrow \quad \Diamond A$	"Ought" implies "can": "You ought to do A" entails "It's possible for you to do A."

Hare's Law: An "ought" entails the corresponding imperative.

Kant's Law: "Ought" implies "can."

Hume's Law: You can't deduce an "ought" from an "is."

Poincaré's Law: You can't deduce an imperative from an "is."

Valid

* 1 $R(\underline{A} \cdot \underline{B})$

[$\therefore R\underline{A}$

* 2 ┌ asm: $\sim R\underline{A}$

3 │ $\therefore O\sim\underline{A}$ {from 2} ← reverse squiggles

* 4 │ $D \therefore (\underline{A} \cdot \underline{B})$ {from 1} ← drop “R”

5 │ $D \therefore \underline{A}$ {from 4}

6 │ $D \therefore \underline{B}$ {from 4}

7 └ $D \therefore \sim\underline{A}$ {from 3} ← drop “O”

8 $\therefore R\underline{A}$ {from 2; 5 contradicts 7}

1. Reverse squiggles.
2. Drop each initial “R,” using a new deontic world each time.
3. Lastly, drop each initial “O” once for each old deontic world. (Never use a new deontic world when you drop “O.”)

[$\therefore \Box(O(\underline{A} \cdot \underline{B}) \supset O\underline{A})$ Valid

* 1 asm: $\sim\Box(O(\underline{A} \cdot \underline{B}) \supset O\underline{A})$

* 2 $\therefore \Diamond\sim(O(\underline{A} \cdot \underline{B}) \supset O\underline{A})$ {from 1}

* 3 W $\therefore \sim(O(\underline{A} \cdot \underline{B}) \supset O\underline{A})$ {from 2}

4 W $\therefore O(\underline{A} \cdot \underline{B})$ {from 3}

* 5 W $\therefore \sim O\underline{A}$ {from 3}

* 6 W $\therefore R\sim\underline{A}$ {from 5}

7 WD $\therefore \sim\underline{A}$ {from 6}

8 WD $\therefore (\underline{A} \cdot \underline{B})$ {from 4}

9 WD $\therefore \underline{A}$ {from 8}

10 $\therefore \Box(O(\underline{A} \cdot \underline{B}) \supset O\underline{A})$ {from 1; 7 contradicts 9}