Galactic Travel

System S5 lets us go from "□A" in any world W1 to "A" in any world W2.	Weaker systems require a suitable travel ticket between the worlds. We get travel tickets when you drop diamonds – and we use them when dropping boxes.

System T	*System S4*	*System B*
We need a *ticket* from W1 to W2.	Like T, but we also can use a *series* of tickets.	Like T, but a ticket also works *backwards*.

	1	$\Box(N \supset \Box N)$	
*	2	$\Diamond N$	
		[$\therefore \Box N$	
*	3	┌ asm: $\sim\Box N$	
*	4	│ $\therefore \Diamond\sim N$ {from 3}	
*	5	│ W $\therefore$ N {from 2}	$\# \Rightarrow$ W
	6	│ WW $\therefore \sim N$ {from 4}	$\# \Rightarrow$ WW
*	7	│ W $\therefore (N \supset \Box N)$ {from 1}	T or any other system
	8	│ W $\therefore \Box N$ {from 5 and 7}	
	9	└ WW $\therefore$ N {from 8}	Need S5
	10	$\therefore \Box N$ {from 3; 6 contradicts 9}	

In dropping "□" (steps 8 to 9), we need to use
a *series* of tickets (and one *backwards*), from W to WW.

	1	□A	
		[∴ □□A	
*	2	┌ asm: ~□□A	
*	3	│ ∴ ◇~□A {from 2}	
*	4	│ W ∴ ~□A {from 3}	# ⇒ W
*	5	│ W ∴ ◇~A {from 4}	
	6	│ WW ∴ ~A {from 5}	W ⇒ WW
	7	└ WW ∴ A {from 1}	Need S4 or S5
	8	∴ □□A {from 2; 6 contradicts 7}	

In dropping "□" (from 1 to 7), we need to use a *series* of tickets, from # to WW.

	1	A		
		[∴ □◇A		
*	2	┌ asm: ~□◇A		
*	3	│ ∴ ◇~◇A	{from 2}	
*	4	│ W ∴ ~◇A	{from 3}	# ⇒ W
	5	│ W ∴ □~A	{from 4}	
	6	└ ∴ ~A	{from 5}	Need B or S5
	7	∴ □◇A	{from 2; 1 contradicts 6}	

In dropping "□" (from 5 to 6), we need to use a ticket *backwards*.

*	1	◇A		
		[∴ □◇A		
*	2	┌ asm: ~□◇A		
*	3	│ ∴ ◇~◇A	{from 2}	
*	4	│ W ∴ ~◇A	{from 3}	# ⇒ W
	5	│ W ∴ □~A	{from 4}	
	6	│ WW ∴ A	{from 1}	# ⇒ WW
	7	└ WW ∴ ~A	{from 5}	Need S5
	8	∴ □◇A	{from 2; 6 contradicts 7}	

In dropping "□" (steps 5 to 7), we need to use a *series* of tickets (and one *backwards*), from W to WW.

Quantified Modal Translations

$\Diamond(\exists x)Ax$ = It's possible for *someone* to be above average.

$\Diamond(x)Ax$ = It's possible for *everyone* to be above average.

$(x)\Diamond Ax$ = It's possible for *anyone* to be above average.

$\Box Fx$ = F is a necessary (essential) property of x.
= x is necessarily F.

$(Fx \cdot \Diamond\sim Fx)$ = F is a contingent (accidental) property of x.
= x is F but could have lacked F.

This ambiguous sentence could have either of two meanings:

"All bachelors are necessarily unmarried."

Simple necessity	**Conditional necessity**
$(x)(Bx \supset \Box Ux)$	$\Box(x)(Bx \supset Ux)$
All bachelors are *inherently unmarriable* – in no possible world would anyone marry them.	It's necessarily true that all bachelors are unmarried. (The meaning of "bachelor" makes this true.)

$\Diamond(\exists x)Ax$ = It's possible for *someone* to be above average.

$\Diamond(x)Ax$ = It's possible for *everyone* to be above average.

$(x)\Diamond Ax$ = It's possible for *anyone* to be above average.

$\Box Fx$ = F is a necessary (essential) property of x.
= x is necessarily F.

$(Fx \cdot \Diamond{\sim}Fx)$ = F is a contingent (accidental) property of x.
= x is F but could have lacked F.

All bachelors are necessarily unmarried = $(x)(Bx \supset \Box Ux)$ or $\Box(x)(Bx \supset Ux)$

```
      1       □(x)x=x    Valid
        [ ∴ (x)□x=x
*     2  ┌ asm: ~(x)□x=x
*     3  │ ∴(∃x)~□x=x    {from 2}
*     4  │ ∴~□a=a    {from 3}
*     5  │ ∴◇~a=a    {from 4}
      6  │ W ∴ ~a=a    {from 5}
      7  │ W ∴ (x)x=x    {from 1}
      8  └ W ∴ a=a    {from 7}
      9  ∴ (x)□x=x    {from 2; 6 contradicts 8}
```

1. Reverse squiggles (modal and quantificational).
2. Drop weak operators (modal and quantificational).
3. Lastly, drop strong operators (modal and quantificational).

All bachelors are necessarily unmarried.
You're a bachelor.
∴ "You're unmarried" is logically necessary.

Valid

1 (x)(Bx ⊃ □Ux)
2 Bu
[∴ □Uu
3 asm: ~□Uu
* 4 ∴ (Bu ⊃ □Uu) {from 1}
5 ∴ □Uu {from 4 and 2}
6 ∴ □Uu {from 3; 3 contradicts 5}

(While this is valid, premise 1 is false.)

Invalid

1 □(x)(Bx ⊃ Ux)
2 Bu
[∴ □Uu
* 3 asm: ~□Uu
* 4 ∴ ◇~Uu {from 3}
5 W ∴ ~Uu {from 4}
6 W ∴ (x)(Bx ⊃ Ux) {from 1}
7 ∴ (x)(Bx ⊃ Ux) {from 1}
* 8 W ∴ (Bu ⊃ Uu) {from 6}
* 9 ∴ (Bu ⊃ Uu) {from 7}
10 W ∴ ~Bu {from 5 and 8}
11 ∴ Uu {from 2 and 9}

	Bu, Uu
W	~Bu, ~Uu

It's possible for anyone to be above average.	$(x)\Diamond Ax$
∴ It's possible for everyone to be above average.	∴ $\Diamond(x)Ax$

This lead into an endless loop.
Using ingenuity, we can devise a refutation:

	a, b
W	Aa, ∼Ab
WW	Ab, ∼Aa

Problem: Our system doesn't recognize the ambiguity here:

The number I'm thinking of is necessarily odd = $\Box On$

I'm thinking of just one number, and it has the necessary property of being odd. (Perhaps true!) $(\exists x)((Tx \cdot \sim(\exists y)(\sim x=y \cdot Ty)) \cdot \Box Ox)$
This is necessary: "I'm thinking of just one number and it is odd." (False!) $\Box(\exists x)((Tx \cdot \sim(\exists y)(\sim x=y \cdot Ty)) \cdot Ox)$

Solution: Analyze "the …" using Russell's theory of descriptions.

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Problem: Our system makes this valid – which it isn't!

$e=n$ $\Box e=e$ $\therefore \Box e=n$	8 is the number I'm thinking of. It's necessary that 8 is 8. ∴ It's necessary that 8 is the number I'm thinking of.

Solution: Analyze "the …"
using Russell's theory of descriptions.

Problem: Our system makes every entity a necessary being! Here's a proof that in every possible world there exists a being who is Gensler:

[$\therefore \Box(\exists x)x=g$ Valid ???

* 1 asm: $\sim\Box(\exists x)x=g$

* 2 $\therefore \Diamond\sim(\exists x)x=g$ {from 1}

* 3 W $\therefore \sim(\exists x)x=g$ {from 2}

4 W $\therefore (x)\sim x=g$ {from 3}

5 W $\therefore \sim g=g$ {from 4} ← ???

6 W $\therefore g=g$ {self-identity rule}

7 $\therefore \Box(\exists x)x=g$ {from 1; 5 contradicts 6}

Solution: Reject the step from 4 to 5. Move to a "free logic" – one that is free of the assumption that individual constants like "g" always refer to existing beings.

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Free logic reformulates the rules for dropping "(∃x)" and "(x)":

$(\exists x)Fx \quad \rightarrow \quad Fa,\ (\exists x)x{=}a$, use a *new* constant	Some existing being is F. ∴ a is F. ∴ a is an existing being.
$(x)Fx,\ (\exists x)x{=}a \quad \rightarrow \quad Fa$, use *any* constant	Every existing being is F. a is an existing being. ∴ a is F.

Our sophisticated system uses “free logic” rules (free of the assumption that constants like “g” refer to existing beings) for dropping “(x)” & “(∃x)”:

(x)Fx, (∃x)x=a → Fa, *use any constant*	Every existing being is F. a is an existing being. ∴ a is F.
(∃x)Fx → Fa, (∃x)x=a, *use a new constant*	Some existing being is F. ∴ a is F. ∴ a is an existing being.

```
      [ ∴ □(∃x)x=g          Valid ???
*  1  ┌ asm: ~□(∃x)x=g
*  2  │ ∴ ◇~(∃x)x=g    {from 1}
*  3  │ W ∴ ~(∃x)x=g    {from 2}
   4  │ W ∴ (x)~x=g    {from 3}
   5  │ W ∴ ~g=g    {from 4}   ←   ???
   6  └ W ∴ g=g    {self-identity rule}
   7  ∴ □(∃x)x=g    {from 1; 5 contradicts 6}
```

Without free logic, every entity (e.g. Gensler) is a necessary being that exists in every possible world. Free logic rejects step 5 in the proof to the left. Now what entities exist can vary from world to world.

Our sophisticated system always uses Russell's theory of descriptions to analyze definite descriptions (terms of the form "the so and so"). This invalidates the following invalid argument:

	e=n		8 is the number I'm thinking of.
	☐e=e		It's necessary that 8 is 8.
∴	☐e=n	∴	It's necessary that 8 is the number I'm thinking of.

This also enables us to express the two senses of "The number I'm thinking of is necessarily odd" (which in naïve QMC is just ☐On):

- This is necessary: "I'm thinking of just one number and it is odd." (False!) ☐(∃x)((Tx • ~(∃y)(~x=y • Ty)) • Ox)
- I'm thinking of just one number, and it has the necessary property of being odd. (Perhaps true!) (∃x)((Tx • ~(∃y)(~x=y • Ty)) • ☐Ox)