Identity Logic

$r=l$	=	Romeo is the lover of Juliet. (identity)
Ir	=	Romeo is Italian. (predication)
$(\exists x)Ix$	=	There are Italians. (existence)

The result of writing a small letter and then "=" and then a small letter is a wff.

Romeo isn't the lover of Juliet = $\sim r=l$

Someone besides Romeo is Italian
Someone who isn't Romeo is Italian = $(\exists x)(\sim x=r \cdot Ix)$

Romeo alone is Italian
Romeo is Italian but no one else is = $(Ir \cdot \sim(\exists x)(\sim x=r \cdot Ix))$

There's at least one Italian = $(\exists x)Ix$

There are at least two Italians = $(\exists x)(\exists y)((Ix \cdot Iy) \cdot \sim x=y)$

Exactly one is dark

$(\exists x)(Dx \cdot {\sim}(\exists y)({\sim}y{=}x \cdot Dy))$

For some x, x is dark and there's no y such that y≠x and y is dark

Exactly two are dark

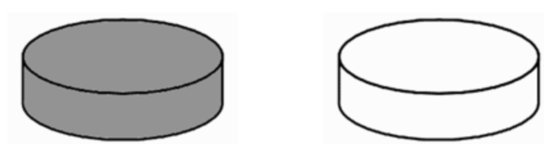
$(\exists x)(\exists y)(((Dx \cdot Dy) \cdot {\sim}x{=}y) \cdot {\sim}(\exists z)(({\sim}z{=}x \cdot {\sim}z{=}y) \cdot Dz))$

For some x and some y, x is dark and y is dark and x≠y and there's no z such that z≠x and z≠y and z is dark

$1 + 1 = 2$

If exactly one being is F
and exactly one being is G
and nothing is F-and-G,
then exactly two beings
are F-or-G.

$$
\begin{gathered}
((((\exists x)(Fx \cdot {\sim}(\exists y)({\sim}y{=}x \cdot Fy)) \\
\cdot\ (\exists x)(Gx \cdot {\sim}(\exists y)({\sim}y{=}x \cdot Gy))) \\
\cdot\ {\sim}(\exists x)(Fx \cdot Gx)) \supset \\
(\exists x)(\exists y)(((Fx \vee Gx) \cdot (Fy \vee Gy)) \cdot ({\sim}x{=}y \\
\cdot\ {\sim}(\exists z)(({\sim}z{=}x \cdot {\sim}z{=}y) \cdot (Fz \vee Gz)))))
\end{gathered}
$$



Identity Principles

Self-identity axiom	$a=a$
Substitute-equals rule	$a=b,\ Fa \quad \rightarrow \quad Fb$

There's more than one being. (pluralism)
∴ It's false that there's exactly one being. (monism)

* 1 (∃x)(∃y)~x=y Valid
[∴ ~(∃x)(y)y=x
* 2 ┌ asm: (∃x)(y)y=x
* 3 │ ∴ (∃y)~a=y {from 1}
4 │ ∴ ~a=b {from 3}
5 │ ∴ (y)y=c {from 2}
6 │ ∴ a=c {from 5}
7 │ ∴ b=c {from 5}
8 └ ∴ a=b {from 6 and 7}
9 ∴ ~(∃x)(y)y=x {from 2; 4 contradicts 8}

Do we need to qualify the substitute-equals rule?

	Jones believes that Lincoln is on the penny.	Bl
	Lincoln is the first Republican president.	l=r
∴	Jones believes that the first Republican president is on the penny.	∴ Br

Relational Logic

Lrj	=	Romeo loves Juliet.
Bxyz	=	x is between y and z.

The result of writing a capital letter and then two or more small letters is a wff.

Juliet loves Romeo	=	Ljr
Juliet loves herself	=	Ljj
Juliet loves Romeo but not Paris	=	$(Ljr \cdot \sim Ljp)$

Everyone loves him/herself	=	$(x)Lxx$
Someone loves him/herself	=	$(\exists x)Lxx$
No one loves him/herself	=	$\sim(\exists x)Lxx$

Someone (everyone, no one) loves Romeo

=

For some (all, no) x, x loves Romeo.

Normally put quantifiers *before* relations.

Romeo loves someone (everyone, no one)

=

For some (all, no) x, Romeo loves x.

Someone loves Romeo = $(\exists x)Lxr$
For some x, x loves Romeo

Everyone loves Romeo = $(x)Lxr$
For all x, x loves Romeo

No one loves Romeo = $\sim(\exists x)Lxr$
It's not the case that, for some x, x loves Romeo

Romeo loves someone = $(\exists x)Lrx$
For some x, Romeo loves x

Romeo loves everyone = $(x)Lrx$
For all x, Romeo loves x

Romeo loves no one = $\sim(\exists x)Lrx$
It's not the case that, for some x, Romeo loves x

Some Montague loves Juliet = $(\exists x)(Mx \cdot Lxj)$
For some x, x is a Montague and x loves Juliet

All Montagues love Juliet = $(x)(Mx \supset Lxj)$
For all x, if x is a Montague then x loves Juliet

Romeo loves some Capulet = $(\exists x)(Cx \cdot Lrx)$
For some x, x is a Capulet and Romeo loves x

Romeo loves all Capulets = $(x)(Cx \supset Lrx)$
For all x, if x is a Capulet then Romeo loves x

Some Montague besides Romeo loves Juliet

$(\exists x)((Mx \cdot {\sim}x{=}r) \cdot Lxj)$

For some x, x is a Montague and x ≠ Romeo and x loves Juliet

Romeo loves all Capulets besides Juliet

$(x)((Cx \cdot {\sim}x{=}j) \supset Lrx)$

For all x, if x is a Capulet and x ≠ Juliet then Romeo loves x

Romeo loves all Capulets who love themselves

$(x)((Cx \cdot Lxx) \supset Lrx)$

For all x, if x is a Capulet and x loves x then Romeo loves x

These have two different relations:

All who know Juliet love Juliet
$(x)(Kxj \supset Lxj)$
For all x, if x knows Juliet then x loves Juliet

All who know themselves love themselves
$(x)(Kxx \supset Lxx)$
For all x, if x knows x then x loves x

Translate these now.

1. God loves Ignatius.
2. Everyone loves God.
3. God loves everyone.
4. All Jesuits love God.
5. God loves some Jesuits.
6. God loves everyone who doesn’t love himself.
7. God loves all Jesuits who don’t love themselves.
8. All Jesuits love themselves.
9. Ignatius loves everyone besides himself.
10. Some Jesuits love some besides themselves.

These have two quantifiers:

Someone loves someone
$(\exists x)(\exists y)Lxy$
For some x and for some y, x loves y

Everyone loves everyone
$(x)(y)Lxy$
For all x and for all y, x loves y

Every Montague hates every Capulet
$(x)(y)((Mx \cdot Cy) \supset Hxy)$
For all x and for all y, if x is a Montague
and y is a Capulet then x hates y

Everyone loves someone.	There's someone who everyone loves.
For all x there's some y, such that x loves y.	There's some y such that, for all x, x loves y.
$(x)(\exists y)Lxy$	$(\exists y)(x)Lxy$

weaker claim
$(x)(\exists y)$

stronger claim
$(\exists y)(x)$

Until you master harder relational translations, go by "baby steps" from English to Loglish to symbols.

Every Capulet loves some Montague
For all x, if x is a Capulet then x loves some Montague
(x)(Cx ⊃ x loves some Montague)
(x)(Cx ⊃ for some y, y is a Montague and x loves y)
$(x)(Cx \supset (\exists y)(My \cdot Lxy))$

Some Capulet loves every Montague
For some x, x is a Capulet and x loves every Montague
(∃x)(Cx · x loves every Montague)
(∃x)(Cx · for all y, if y is a Montague then x loves y)
$(\exists x)(Cx \cdot (y)(My \supset Lxy))$

There's an unloved lover
For some x, x is unloved (no one loves x) and
x is a lover (x loves someone)
$(\exists x)(\sim(\exists y)Lyx \cdot (\exists y)Lxy)$

Everyone loves a lover
For all x, if x is a lover (x loves someone) then everyone loves x
$(x)((\exists y)Lxy \supset (y)Lyx)$

Romeo loves all and only those who don't love themselves
For all x, Romeo loves x if and only if x doesn't love x
$(x)(Lrx \equiv \sim Lxx)$

All who know any person love that person
For all x and all y, if x knows y then x loves y
$(x)(y)(Kxy \supset Lxy)$

Reflexive / Irreflexive

Everyone loves himself	=	(x)Lxx
No one loves himself	=	(x)~Lxx

Symmetrical / Asymmetrical

Universally, if x loves y then y loves x [does not love x]	=	$(x)(y)(Lxy \supset Lyx)$
	=	$(x)(y)(Lxy \supset {\sim}Lyx)$

Transitive / Intransitive

Universally, if x loves y and y loves z, then x loves z [does not love z]	=	$(x)(y)(z)((Lxy \cdot Lyz) \supset Lxz)$
	=	$(x)(y)(z)((Lxy \cdot Lyz) \supset {\sim}Lxz)$

Translate these now.

1. Some Jesuits love everyone.
2. No one loves all Franciscans.
3. All Jesuits love someone.
4. There is someone that all Jesuits love.
5. There is some Franciscan that everyone loves.
6. Some Franciscans love all Jesuits.
7. No Jesuits love all Franciscans.
8. Ignatius loves all and only those who don’t love themselves.

Translate these now.

1. Every Capulet loves some Montague.
2. Universally, if x knows y then x loves y.
3. There is an unloved lover.
4. Everyone loves all lovers.
5. Some Jesuits besides Ignatius love God.

1. Every Capulet loves some Montague.

   $(x)(Cx \supset (\exists y)(My \cdot Lxy))$

2. Universally, if x knows y then x loves y.

   $(x)(y)(Kxy \supset Lxy)$

3. There is an unloved lover.

   $(\exists x)(\sim(\exists y)Lyx \cdot (\exists y)Lxy)$

4. Everyone loves all lovers.

   $(x)((\exists y)Lxy \supset (y)Lyx)$

5. Some Jesuits besides Ignatius love God.

   $(\exists x)((Jx \cdot \sim x=i) \cdot Lxg)$

Hans loves Olga	=	Lho
Hans loves someone	=	$(\exists x)Lhx$
Hans loves some Russian	=	$(\exists x)(Rx \cdot Lhx)$
Someone loves some Russian	=	$(\exists x)(\exists y)(Ry \cdot Lxy)$
Some German loves some Russian	=	$(\exists x)(Gx \cdot (\exists y)(Ry \cdot Lxy))$
Everyone loves some Russian	=	$(x)(\exists y)(Ry \cdot Lxy)$
Every German loves some Russian	=	$(x)(Gx \supset (\exists y)(Ry \cdot Lxy))$

Hans loves Olga	=	Lho
Hans loves everyone	=	???
Hans loves every Russian	=	???
Someone loves every Russian	=	???
Some German loves every Russian	=	???
Everyone loves every Russian	=	???
Every German loves every Russian	=	???

Hans loves Olga	=	Lho
Hans loves everyone	=	$(x)Lhx$
Hans loves every Russian	=	$(x)(Rx \supset Lhx)$
Someone loves every Russian	=	$(\exists x)(y)(Ry \supset Lxy)$
Some German loves every Russian	=	$(\exists x)(Gx \cdot (y)(Ry \supset Lxy))$
Everyone loves every Russian	=	$(x)(y)(Ry \supset Lxy)$
Every German loves every Russian	=	$(x)(Gx \supset (y)(Ry \supset Lxy))$

	1	(x)Lxx	**Valid**
		[∴ (x)(∃y)Lxy	
*	2	┌ asm: ~(x)(∃y)Lxy	
*	3	│ ∴ (∃x)~(∃y)Lxy {from 2}	
*	4	│ ∴ ~(∃y)Lay {from 3}	
	5	│ ∴ (y)~Lay {from 4}	
	6	│ ∴ ~Laa {from 5}	
	7	└ ∴ Laa {from 1}	
	8	∴ (x)(∃y)Lxy {from 2; 4 contradicts 6}	

Relational proofs are often tricky, even though they use no new inference rules. When you have a string of quantifiers, as in lines 2 and 3 above, work on one at a time, starting from the outside. Drop only *initial* quantifiers!

	1	$(x)(\exists y)Lxy$	
		[$\therefore$ Laa	
	2	asm: $\sim$Laa	
*	3	$\therefore (\exists y)Lay$	{from 1}
	4	$\therefore$ Lab	{from 3}
	5	$\therefore (\exists y)Lby$	{from 1}

Invalid

a, b

$\boxed{\sim Laa,\ Lab,\ Lba}$

➔ get c, d, . . .

"$(x)(\exists y)$" often generates an endless loop:

Since everyone loves someone $(x)(\exists y)Lxy$	a loves someone, call this person b b loves someone, call this person c c loves someone, call this person d …

If you see an endless loop coming, break out of it (usually stop at two constants) and *invent a refutation*.

	1	(x)Lxx	
		[∴ (∃x)(y)Lyx	
*	2	asm: ~(∃x)(y)Lyx	
	3	∴ (x)~(y)Lyx {from 2}	
	4	∴ Laa {from 1}	
*	5	∴ ~(y)Lya {from 3}	
*	6	∴ (∃y)~Lya {from 5}	
	7	∴ ~Lba {from 6}	
	8	∴ Lbb {from 1}	
*	9	∴ ~(y)Lyb {from 3}	
	10	∴ (∃y)~Lyb {from 9} . . .	➔ get c, d, . . .

Invalid

a, b

Laa, Lbb
~Lba, ~Lab

> If you see an endless loop coming, break out of it and invent your own refutation.

Alonzo Church's Theorem (1931)

The problem of determining validity in relational logic cannot be reduced to an algorithm (a finite mechanical procedure).

Russell's theory of definite descriptions

The king of France is bald
$(\exists x)((Kx \cdot \sim(\exists y)(\sim y{=}x \cdot Ky)) \cdot Bx)$
There's exactly one king of France, and he's bald
For some x, x is king of France and there's no y such that:
$y \neq x$ and y is king of France and x is bald

This symbolizes the English statement better than "Bk," since:

- the statement can be false for three reasons (there's no king of France, there's more than one, or there's just one but with hair) and
- we more easily avoid the metaphysical error of thinking that "the round square" refers to an existing thing that isn't real.